Defaults ในระยะสั้นและในระยะยาว (Assessing Default Probabilities in the Short Run and in the Long Run)

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ภาควิชาการเงิน คณะพาณิชยศาสตร์และการบัญชี

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Topics

- Motivation
- Assumptions
- Computation Process
- Examples of Transition Probability Matrix (TPM) for 6 months and 2 years

Motivation

- The time interval within which a Transition Probability Matrix (TPM) is estimated is typically one year.
- However, for investment (e.g. bank deposits and corporate bonds) and valuation purposes (e.g. valuing a default swap), we need a TPM for a period shorter or longer than a year.
- The number of transition observations within a shorter period is too small for a reliable TPM to be estimated.

Motivation

- For longer period, one obvious way could be to reestimate the TPM by observing the longer period of rating changes instead.
  - It would mean we need to go through the whole process of TPM calculation for each particular time horizon.
  - Due to infrequent rating announcements, we could end up with exactly the same TPM for 14-month and 15-month time horizon.
- One solution is to find a transformation method using the existing one-year TPM to find a shorter and longer TPM.
  - Efficient but need to understand the assumptions behind.

Assumptions

- Let ***P*** be a time-homogeneous (i.e. time-invariant) Markov transition matrix.
  - One-step transition probabilities remain constant over time.
- The Markov property is an assumption on the conditional probability distribution that allows the future rating to be independent of past rating history (or time already spent in the rating).
- The possibility than an obligor recovers from the default state is ignored.
  - Once an obligor reaches the default state K, it is assumed to remain there forever.

Assumptions

- If we can find a valid/exact Generator Matrix (GM)—***Q*** defined by

$$Q = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1K} \\ q_{21} & q_{22} & \cdots & q_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ q_{K1} & q_{K2} & \cdots & q_{KK} \end{bmatrix}$$

  - which satisfies the following properties

$$q(i, j \neq i) \geq 0$$

$$q(i,i) = -\sum_{j \neq i}^{K} q(i, j)$$

  - such that $Exp\{Q\} = P$
  - Then we can set $P(h) = Exp\{hQ(h)\}$ to obtain matrices for any time $h \geq 0$.
  - See *Israel et al. (2001)* for more details and proofs.

Assumptions

- The holding time of an obligor in rating grade i--S_i before migrating from it is exponentially distributed with a parameter q_i.

$$S_i \sim Exp\left(-q_i h\right)$$

$$q_i = \sum_{j=1,\, j\neq i}^{K} q_{ij} = -q\left(i,i\right)$$

- Given a transition in rating i, the conditional probability of an obligor migrating to a new rating grade j is multinomially distributed with q_{ij}/q_i.

Assumptions

- The default state *K* is assumed to occur in the long run, regardless of the initial rating agencies.

$$\lim_{h \to \infty} Exp\{hQ(h)\} \to D$$

$$D = \begin{bmatrix} 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

- See *Inamura (2006)* for more details.

Computation Process

- Choose an estimation/adjustment method to obtain a valid/exact Generator Matrix (GM)—***Q***. *Inamura (2006)* showed that there are at least 5 methods:
  - Diagonal Adjustment** (Recommended by Professor Dr.Anya Khantavit)
  - Weighted Adjustment
  - Quasi-optimization methodology
  - Expectation-maximization (EM) algorithm
  - Markov chain Monte Carlo (MCMC) estimation method

Computation Process

- For one-year TPM, *h* is assumed to equal to 1 (*h* = 1).
- Choose the interested *h*** such as 6 month (*h*** = 6/12).
- Calculate the following equations:

$$P(h) = Exp\{hQ(h)\}$$

$$Exp(hQ) = \sum_{k=0}^{\infty} \frac{(hQ)^k}{k!}$$

- Finding reliable and accurate methods to compute the matrix exponential is difficult, and there several approximation methods.

Finding Tools for Computation

- Excel
  - Add-in called “MATRIX 2.3 - Matrix and Linear Algebra functions for EXCEL”
  - http://digilander.libero.it/foxes/SoftwareDownload.htm
- Gauss
  - Source Code called “Mlib1”
  - http://www.thierry-roncalli.com/#gauss
- Matlab
  - A function called “expm”
  - http://www.mathworks.com/help/techdoc/ref/expm.html

Examples using Excel

- Install the add-in “MATRIX 2.3”
- Use Excel template called “TPM_Suluck”

Excel Template

Transition Probability Matrix (TPM)

เป็นโปรแกรมที่ใช้คำนวณ TPM ณ เวลา *h* ใดๆ จากข้อมูล Generator Matrix ที่มี โดยมีขั้นตอนดังนี้
1. หากมีข้อมูล Generator Matrix ที่มีการปรับปรุงใหม่ให้ทำการคัดลอกสู่เซลล์ B24:I31
2. ระบุระยะเวลา *h* ที่ต้องการ โดย *h* มีหน่วยเป็นปี ที่เซลล์ B8

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หมายเหตุ ผู้ใช้ต้องติดตั้งโปรแกรม Add-in ที่ชื่อว่า Matrix and Linear Algebra for Excel v.23 ซึ่งสามารถ Download ได้โดยไม่เสียค่าใช้จ่ายจาก http://digilander.libero.it/foxes/SoftwareDownload.htm

เวลา (*h*)	1.0000	ปี

Transition Probablity Matrix

อันดับ ณ ต้นปี	อับดับ ณ สิ้นเวลา 1.0000 ปี							
	AAA	AA	A	BBB	BB	B	CCC/C	Default
AAA	85.5400%	13.6570%	0.6477%	0.0410%	0.0516%	0.0193%	0.0321%	0.0113%
AA	0.5935%	90.1462%	8.5154%	0.5523%	0.0624%	0.0833%	0.0208%	0.0260%
A	0.0149%	2.8998%	92.8949%	3.7051%	0.1414%	0.0633%	0.0076%	0.2730%
BBB	0.0026%	0.0997%	6.2398%	87.6931%	2.7378%	0.7523%	0.0411%	2.4336%
BB	0.0118%	0.0321%	0.3726%	10.1796%	77.0000%	4.3030%	0.4477%	7.6533%
B	0.0006%	0.0434%	0.1650%	0.3977%	5.8448%	78.3813%	7.4604%	7.7069%
CCC/C	0.0001%	0.0079%	0.2242%	0.3428%	0.9676%	15.1932%	51.2391%	32.0250%
Default	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	100.0000%

Generator Matrix

อันดับ ณ ต้นปี	อับดับ ณ สิ้นปี							
	AAA	AA	A	BBB	BB	B	CCC/C	Default
AAA	-0.156721	0.155600	0.000000	0.000000	0.000574	0.000098	0.000449	0.000000
AA	0.006759	-0.105746	0.093024	0.004247	0.000573	0.000902	0.000240	0.000000
A	0.000059	0.031723	-0.076555	0.040983	0.000954	0.000515	0.000061	0.002261
BBB	0.000024	0.000000	0.069221	-0.134748	0.033021	0.008192	0.000000	0.024290
BB	0.000143	0.000313	0.000000	0.124025	-0.265605	0.054681	0.003637	0.082807
B	0.000000	0.000476	0.001661	0.000000	0.075216	-0.257932	0.117845	0.062734
CCC/C	0.000000	0.000000	0.002832	0.004243	0.005620	0.240104	-0.685010	0.432211
Default	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

หมายเหตุ: เป็น Matrix ที่ได้จากผลงานวิจัยเรื่อง "การกำหนดเมทริกซ์ความน่าจะเป็นของการเปลี่ยนแปลงอันดับเครดิตของตราสารหนี้ไทยซึ่งมีคุณสมบัติสอดคล้องเต็มที่กับทฤษฎีการเงิน" โดย ศ.ดร.อัญญา ขันธวิทย์ ปี พ.ศ.2554

TPM for 6 Months

Transition Probablity Matrix								
อันดับ ณ ต้นปี						**อันดับ ณ สิ้นเวลา**	**0.5000**	**ปี**
	AAA	AA	A	BBB	BB	B	CCC/C	Default
AAA	92.4754%	7.2873%	0.1711%	0.0098%	0.0271%	0.0076%	0.0189%	0.0029%
AA	0.3166%	94.8976%	4.4488%	0.2461%	0.0299%	0.0433%	0.0111%	0.0065%
A	0.0053%	1.5160%	96.3138%	1.9475%	0.0602%	0.0288%	0.0034%	0.1249%
BBB	0.0012%	0.0263%	3.2846%	93.5651%	1.5025%	0.3926%	0.0118%	1.2159%
BB	0.0065%	0.0154%	0.1007%	5.6144%	87.6553%	2.4225%	0.2101%	3.9752%
B	0.0002%	0.0227%	0.0819%	0.1108%	3.3111%	88.2360%	4.6728%	3.5646%
CCC/C	0.0000%	0.0022%	0.1252%	0.1862%	0.4087%	9.5185%	71.2695%	18.4896%
Default	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	100.0000%

Differences between $TPM_{1\text{-year}}$ and $TPM_{6\text{-month}}$

Transition Probablity Matrix % Change from TPM 1-year								
อันดับ ณ ต้นปี						อับดับ ณ สิ้นเวลา	0.5000	ปี
	AAA	AA	A	BBB	BB	B	CCC/C	Default
AAA	8.11%	-46.64%	-73.58%	-76.20%	-47.45%	-60.94%	-41.27%	-73.98%
AA	-46.65%	5.27%	-47.76%	-55.43%	-52.19%	-48.05%	-46.53%	-75.00%
A	-64.18%	-47.72%	3.68%	-47.44%	-57.44%	-54.45%	-55.12%	-54.25%
BBB	-53.51%	-73.66%	-47.36%	6.70%	-45.12%	-47.81%	-71.27%	-50.04%
BB	-45.08%	-52.10%	-72.98%	-44.85%	13.84%	-43.70%	-53.08%	-48.06%
B	-72.55%	-47.78%	-50.36%	-72.14%	-43.35%	12.57%	-37.36%	-53.75%
CCC/C	-79.01%	-71.57%	-44.17%	-45.69%	-57.76%	-37.35%	39.09%	-42.27%
Default	NA	NA	NA	NA	NA	NA	NA	0.00%

- Change = $\{TPM_{6\text{-month}}(i,j) - TPM_{1\text{-year}}(i.j)\}/TPM_{1\text{-year}}(i,j)$

TPM for 2 years

Transition Probablity Matrix								
อันดับ ณ ต้นปี	อันดับ ณ สิ้นเวลา 2.0000 ปี							
	AAA	AA	A	BBB	BB	B	CCC/C	Default
AAA	73.2521%	24.0124%	2.3215%	0.1759%	0.0958%	0.0509%	0.0485%	0.0430%
AA	1.0440%	81.5919%	15.6253%	1.3047%	0.1369%	0.1559%	0.0370%	0.1041%
A	0.0439%	5.3135%	86.7735%	6.7217%	0.3473%	0.1460%	0.0184%	0.6357%
BBB	0.0064%	0.3598%	11.2883%	77.4143%	4.5622%	1.3775%	0.1259%	4.8655%
BB	0.0198%	0.0780%	1.2791%	16.7977%	59.8250%	6.8309%	0.8994%	14.2701%
B	0.0019%	0.0809%	0.3496%	1.2874%	9.1650%	62.8244%	9.6965%	16.5943%
CCC/C	0.0004%	0.0249%	0.3739%	0.6436%	2.1386%	19.7378%	27.3924%	49.6883%
Default	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	100.0000%

Differences between $TPM_{1\text{-year}}$ and $TPM_{2\text{-year}}$

Transition Probablity Matrix % Change from TPM 1-year								
อันดับ ณ ต้นปี					อับดับ ณ สิ้นเวลา		2.0000	ปี
	AAA	AA	A	BBB	BB	B	CCC/C	Default
AAA	-14.37%	75.82%	258.43%	329.09%	85.83%	163.12%	51.07%	280.61%
AA	75.90%	-9.49%	83.50%	136.23%	119.26%	87.15%	77.65%	299.96%
A	194.65%	83.24%	-6.59%	81.42%	145.58%	130.55%	143.18%	132.83%
BBB	143.31%	260.92%	80.91%	-11.72%	66.64%	83.10%	206.65%	99.93%
BB	67.11%	143.49%	243.30%	65.01%	-22.31%	58.75%	100.89%	86.46%
B	234.05%	86.32%	111.91%	223.70%	56.81%	-19.85%	29.97%	115.32%
CCC/C	368.36%	215.52%	66.76%	87.73%	121.02%	29.91%	-46.54%	55.15%
Default	NA	NA	NA	NA	NA	NA	NA	0.00%

- Change = $\{TPM_{2\text{-year}}(i,j) - TPM_{1\text{-year}}(i.j)\}/TPM_{1\text{-year}}(i,j)$

References

- Inamura, Y., 2006, Estimating continuous time transition matrices from discretely observed data, Bank of Japan Working Paper No. 06-E-07-April 2006, Tokyo.
- Israel, R., J. Rosenthal, and J. Wei, 2001, Finding generator for Markov chains via empirical transition matrices, with applications to credit ratings, ***Mathematical Finance*** 11, 245-265.
- Jarrow, R., D. Lando, and S. Turnbull, 1997, A Markov model for the term structure of credit risk spreads, ***Review of Financial Studies*** 10, 481-523.

Q & A